

Ficha 7. Ex 8

$$F(t) = \int_0^1 \sin(tx^2 + x^3) dx \quad F'(0) = ?$$

$$\begin{aligned} F'(t) &= \int_0^1 \frac{\partial}{\partial t} (\sin(tx^2 + x^3)) dx \\ &= \int_0^1 x^2 \cos(tx^2 + x^3) dx \end{aligned}$$

$$F'(0) = \frac{1}{3} \int_0^1 3x^2 \cos(x^3) dx = \frac{1}{3} \left[\sin(x^3) \right]_0^1 = \frac{\sin(1)}{3}$$

Ficha 8:

$$2. \begin{cases} u = x+y + \sin(x-y) \\ v = 1 + \log(1+xy) - x \end{cases} \quad \begin{array}{l} u=u(x,y) \\ v=v(x,y) \end{array}$$

Motivação: num viz. de $(u, v) = (2, \log 2)$ e num viz de $(x, y) = (1, 1)$, (x, y) é função de (u, v) e calcule $\frac{\partial y}{\partial v}(2, \log 2)$.

Desejo invertar $(u(x, y), v(x, y))$:

$$(x, y) = (1, 1) \leftrightarrow (u, v) = (2, \log 2)$$

- $u(x, y), v(x, y)$ são de classe C^1 ✓

$$\bullet \quad D_{x,y}(u,v) = \begin{bmatrix} 1 + \cos(x-y) & 1 - \cos(x-y) \\ \frac{y}{1+xy} - 1 & \frac{x}{1+xy} \end{bmatrix}$$

Quando $x, y = 1$,

$$D_{x,y}(u,v) = \begin{bmatrix} 2 & 0 \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \quad \det = 1 \neq 0$$

Pelo T.F. Inverso, posso escrever $x = x(u,v)$ e $y = y(u,v)$

Mais viz. de $(u,v) = (2, \log 2)$,

$$D_{x,y}(x,y)(2, \log 2) = (D_{x,y}(u,v)(1,1))^{-1}$$

$$= \begin{bmatrix} 2 & 0 \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}^{-1} = \frac{1}{1} \begin{bmatrix} u & v \\ \frac{1}{2} & 0 \\ \frac{1}{2} & 1 \end{bmatrix} \underset{\det}{\uparrow} \underset{\frac{\partial y}{\partial v}}{\downarrow}$$

3. $F = y \sin(x+y) = 0$ define implicitamente x como função de y mais viz. de $(0,\pi)$. $\frac{\partial x}{\partial y}(\pi)$?

- $F \in C^1 \checkmark$
- $F(0, \pi) = \pi \sin \pi = 0 \checkmark$
- $\det D_x F \neq 0 ?$

$$\det D_x F(0, \pi) = y \cos(x+y) \Big|_{(0, \pi)} = \pi \cos(\pi) = -\pi \neq 0$$

Pelo T. F. Implícito, $x = u(y)$ na vizinhança de $(0, \pi)$

$$\frac{\partial u}{\partial y}(\pi) = - \left(D_x F(0, \pi) \right)^{-1} \cdot D_y F(0, \pi)$$

$$= - \frac{1}{-\pi} \cdot -\pi = -1$$

$\triangleq \frac{\partial F}{\partial y}(0, \pi) = \left. \frac{\partial F}{\partial y}(x, y) \right|_{(x, y) = (0, \pi)} = \sin \pi + \pi \cos \pi = -\pi$

$F = y \sin(x+y)$

Podemos resolver $F=0$ explicitamente:

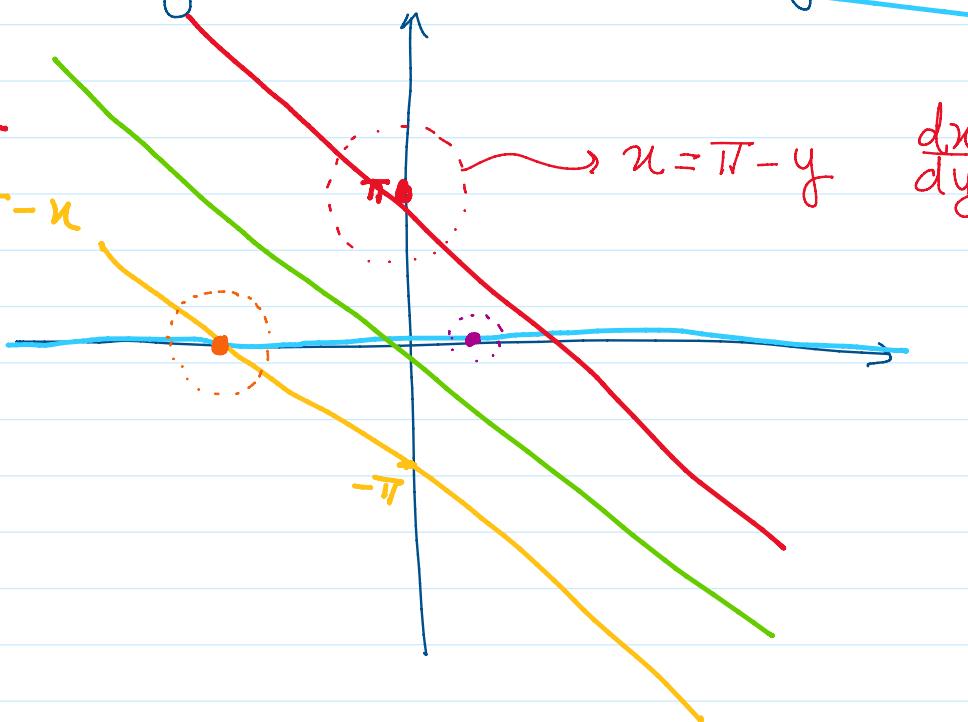
$$y \sin(x+y) = 0 \Leftrightarrow \sin(x+y) = 0 \vee y = 0$$

$$\Leftrightarrow x+y = k\pi, k \in \mathbb{Z} \text{ ou } y = 0$$

$k=0 \quad y = -x$

$k=1 \quad y = \pi - x$

$k=-1 \quad y = -\pi - x$



5.

S: $\begin{cases} y^2 + z^2 = x^2 + 1 \\ y^2 + \sin x + \sin z = 1 \end{cases}$

$$\left\{ \begin{array}{l} y^2 + \sin x + \sin z = 1 \end{array} \right.$$

? Na viz. de $(0, 1, 0)$, S é o gráfico de
uma função de uma variável com valores em \mathbb{R} ?

\Leftrightarrow ? implícite c/ duas variáveis dependentes?

$$F_1 = y^2 + z^2 - x^2 - 1$$

$$\left\{ \begin{array}{l} F_1 = 0 \end{array} \right.$$

$$F_2 = y^2 + \sin x + \sin z - 1$$

$$\left\{ \begin{array}{l} F_2 = 0 \end{array} \right.$$

- $F_1, F_2 \in C^1$ ✓ $F = (F_1, F_2)$

- $\left\{ \begin{array}{l} F_1(0, 1, 0) = 0 \\ F_2(0, 1, 0) = 0 \end{array} \right. \Rightarrow \left\{ \begin{array}{l} 1 + 0 - 0 - 1 = 0 \\ 1 + 0 + 0 - 1 = 0 \end{array} \right. \checkmark$

$$D F = \begin{bmatrix} -2x & 2y & 2z \\ \cos x & 2y & \cos z \end{bmatrix}$$

$$DF(0, 1, 0) = \begin{bmatrix} 0 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

Se fizer x, z em função de y :

- $\det D_{x,z} F = \det \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = 0 \quad X$

Se fizer y, z em função de x :

de fazer y, z em função de x :

- $\det D_{y,z} F = \det \begin{bmatrix} 2 & 0 \\ 2 & 1 \end{bmatrix} = 2 \neq 0 \quad \checkmark$

Pelo TF implicado, posso escrever $(y, z) = (y(x), z(x))$
nem vez de $(0, 1, 0)$,

$$(y(0), z(0)) = (1, 0)$$

$$\begin{aligned} D_x^{(y,z)} &= \begin{bmatrix} y'(x) \\ z'(x) \end{bmatrix} \Big|_{x=0} = - (D_{y,z} F(0,1,0))^{-1} \cdot D_x F(0,1,0) \\ &= - \begin{bmatrix} 2 & 0 \\ 2 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \\ &= -\frac{1}{2} \begin{bmatrix} 1 & 0 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \end{aligned}$$

OU

$$F_1(x, y(x), z(x)) = 0, \quad F_2(x, y(x), z(x)) = 0$$

$$\left\{ \begin{array}{l} y(x)^2 + z(x)^2 = x^2 + 1 \\ y(x)^2 + \sin x + \sin(z(x)) = 1 \end{array} \right.$$

Fazendo a derivada em x

$$\left\{ \begin{array}{l} 2y(x) \cdot y'(x) + 2z(x) \cdot z'(x) = 2x \\ 2y(x) \cdot y'(x) + \cos x + \cos(z(x)) z'(x) = 0 \end{array} \right.$$

$$n=0 \quad (y, z) = (1, 0)$$

$$\left\{ \begin{array}{l} 2y'(0) + 0 = 0 \\ 2y'(0) + 1 + 1 \cdot z'(0) = 0 \end{array} \right. \Rightarrow \left\{ \begin{array}{l} y'(0) = 0 \\ z'(0) = -1 \end{array} \right.$$